# STAT 510: Homework 01

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Due: Monday, February 8, 11:59 PM

## Exercise 1 (Independent Events)

Let $A$ and $B$ be independent events. Show that the following are independent:

- $A^c$ and $B$
- $A$ and $B^c$
- $A^c$ and $B^c$

## Exercise 2 (Conditional Probability with Cards)

(Based on **LW** 1.12) Suppose we have three cards:

- The first is green on both sides.
- The second is red on both sides.
- The third is green on one side and red on the other.

We choose a card at random and we see one side. (The side we see is also random.) If the side we see is green, what is the probability that the other side is also green?

Many people intuitively answer 1/2. Calculate the correct answer.

## Exercise 3 (Discrete Distributions)

Given:

- $X \sim \text{Pois}(\lambda = 3.2)$
- $Y \sim \text{Binom}(n = 20, p = 0.3)$
- $Z \sim \text{Geom}(p = 0.4)$

Here, we assume the probability mass function of $Z$ is given by

$$f(z) = (1 - p)^{z-1}p, \quad z = 1, 2, 3, \ldots$$

**Note:** This is *not* the same parameterization of the geometric distribution that R uses by default.

Calculate:

- $P[1 \leq X \leq 4]$.
- $P[Y > 4]$.
- $P[Z > 3]$.

## Exercise 4 (Bayes' Theorem)

Given:

- $\text{P}(Y = A) = 0.09$
- $\text{P}(Y = B) = 0.18$

- $\text{P}(Y = C) = 0.73$
- $X \mid Y = A \sim \text{Poisson}(\lambda_A = 4.25)$
- $X \mid Y = B \sim \text{Poisson}(\lambda_B = 6.14)$
- $X \mid Y = C \sim \text{Poisson}(\lambda_C = 8.51)$

Calculate $\text{P}(Y = B \mid X = 3)$.

### Exercise 5 (Normal Distribution)

(Based on **LW** 2.18) Let $X \sim \text{Normal}(\mu = 2.5, \sigma = 3.2)$.

- Calculate $P(X < 4)$.
- Calculate $P(X > 2)$.
- Find $x$ such that $P(X > x) = 0.05$.
- Calculate $P(X = 4)$.
- Calculate $P(0 < X \leq 3)$.

### Exercise 6 (Single Variable Transformation)

(Based on **LW** 2.4) Let $X$ have the probability density function

$$f_X(x) = \begin{cases} 1/4 & 0 < x < 1 \\ 3/8 & 3 < x < 5 \\ 0 & \text{otherwise} \end{cases}$$

Define $Y = 1/X$. Find the probability density function of $Y$.

### Exercise 7 (Conditional Distribution)

(Based on **LW** 2.17) Given

$$f_{X,Y}(x, y) = \begin{cases} c(x + y)^2 & 0 < x < 1 \text{ and } 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find $P(X < 0.5 \mid Y = 0.5)$.

### Exercise 8 (Conditional Poissons)

(Based on **LW** 2.16) Let $X \sim \text{Poisson}(\lambda)$ and $Y \sim \text{Poisson}(\mu)$ and assume that $X$ and $Y$ are independent. Find the distribution of $X$ given that $X + Y = n$.

### Exercise 9 (Difference Distribution)

Let $X \sim \text{Uniform}(0, 1)$ and $Y \sim \text{Uniform}(0, 1)$ be independent. Find the probability density function of $X - Y$.

### Exercise 10 (Ratio Distribution)

Let $X \sim \text{Uniform}(0, 1)$ and $Y \sim \text{Uniform}(0, 1)$ be independent. Find the probability density function of $X/Y$.

### Exercise 11 (Simulation Study)

Use a computer experiment to verify your results to Exercise 9 and Exercise 10. For each, do the following:

- Generate a vector $x = (x_1, x_2, \ldots x_{1000})$ from a $\text{Uniform}(0, 1)$ distribution.
- Generate a vector $y = (y_1, y_2, \ldots y_{1000})$ from a $\text{Uniform}(0, 1)$ distribution.

- Define $z = x - y$ or $z = x/y$.
- Plot a histogram of $z$. (Be sure to use density histogram, not frequency.)
- Overlay the true density that you calculated.

You may use any computational tool of your choice. Where possible, please supply your code, and of course, the two histograms. If you are using `R`, we recommend using `breaks = 1000`, `xlim = c(0, 25)`, and `ylim = c(0, 0.55)` for the plot of $z = x/y$. (Otherwise, it may be difficult to see that your density matches the histogram.)